☺ Chapter 4 Notes ☺

4.6 – Dilations and the Absolute-Value Family

Objectives:

1. Define absolute value and its notation and use it to model distance.
2. Define the parent absolute-value function, $y=|x|$, and the absolute-value family, $\frac{y-k}{b}=\left|\frac{x-h}{a}\right|$
3. Calculate horizontal and vertical scale factors from points on the image of a graph.
4. Apply horizontal and vertical dilations to functions in general.

absolute value A number's distance from 0 on the number line. The absolute value of a number gives its size, or magnitude, whether the number is positive or negative. The absolute value of a number x is shown as $|x|$. For example, $|-9|=9$ and $|4|=4$. (418)

Find the following:

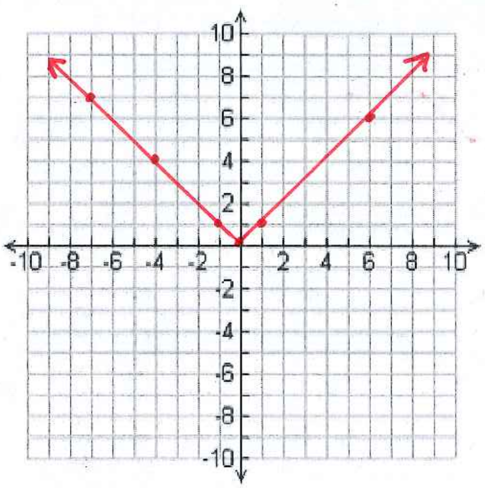
$|-3.5|=$ 3.5

$|0|=$ 0

$\left|\frac{1}{3}\right|=$ $\frac{1}{3}$

absolute-value function The function $f(x)=|x|$, which gives the absolute value of a number. The absolute-value function is defined by two rules: If $x \geq 0$, then $f(x)=x$. If $x<0$, then $f(x)=-x$. (420)

Fill out the table for the values of the absolute value function $y=|x|$. Then graph the parent function.



X	Y
-7	7
-4	4
-1	1
0	0
1	1
6	6

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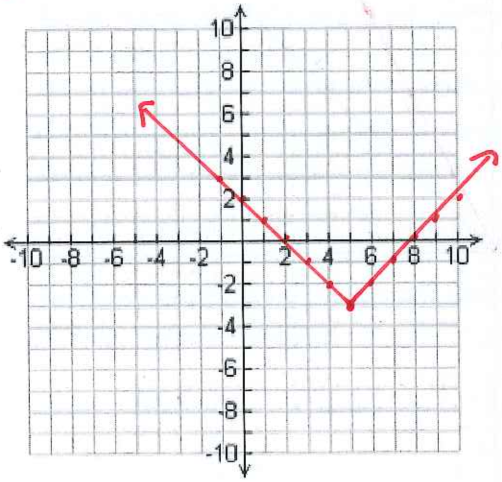
Example 1: Describe the following transformations. Then graph the equation.

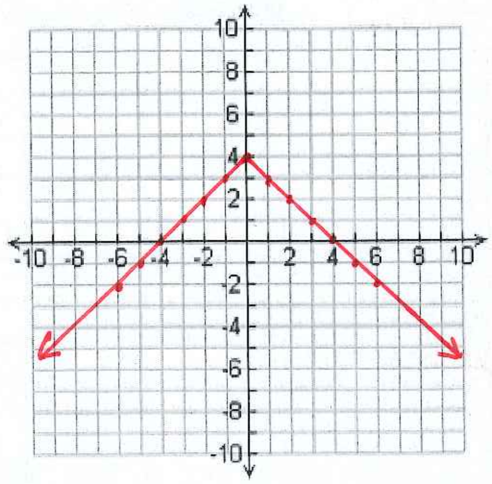
$y = |x-5| - 3$

HORIZONTAL TRANSLATION RIGHT 5
VERTICAL TRANSLATION DOWN 3



$y = -|x| + 4$

VERTICAL REFLECTION ACROSS X
VERTICAL TRANSLATION UP 4



Example 2: Solve each equation for y.

]**a.** $\frac{y}{2} = \left|\frac{x}{4}\right|$

$y = 2\left|\frac{x}{4}\right|$

b. $\frac{y-3}{2} = (x+1)^2$

$y - 3 = 2(x+1)^2$
$+3 \qquad +3$
$y = 2(x+1)^2 + 3$

c. $\frac{y+1}{-3} = \sqrt{x} + 2$

$y + 1 = -3(\sqrt{x} + 2)$
$y + 1 = -3\sqrt{x} - 6$
$-1 \qquad -1$
$y = -3\sqrt{x} - 7$

d. $(y-2)^2 = x$

$y - 2 = \pm\sqrt{x}$
$+2 \qquad +2$
$y = \pm\sqrt{x} + 2$
$y = 2 + \sqrt{x} \quad y = 2 - \sqrt{x}$

e. $\left(\frac{y}{4}\right)^2 = x + 1$

$\frac{y}{4} = \pm\sqrt{x+1}$
$\frac{y}{4} = \sqrt{x+1} \quad \frac{y}{4} = -\sqrt{x+1}$
$y = 4\sqrt{x+1} \quad y = -4\sqrt{x+1}$

f. $\left(\frac{y+5}{9}\right)^2 = x - 4$

$\frac{y+5}{9} = \pm\sqrt{x-4}$
$\frac{y+5}{9} = \sqrt{x-4} \quad \frac{y+5}{9} = -\sqrt{x-4}$
$y + 5 = 9\sqrt{x-4} \quad y + 5 = -9\sqrt{x-4}$
$y = 9\sqrt{x-4} - 5 \quad y = -9\sqrt{x-4} - 5$
$y = -5 + 9\sqrt{x-4} \quad y = -5 - 9\sqrt{x-4}$

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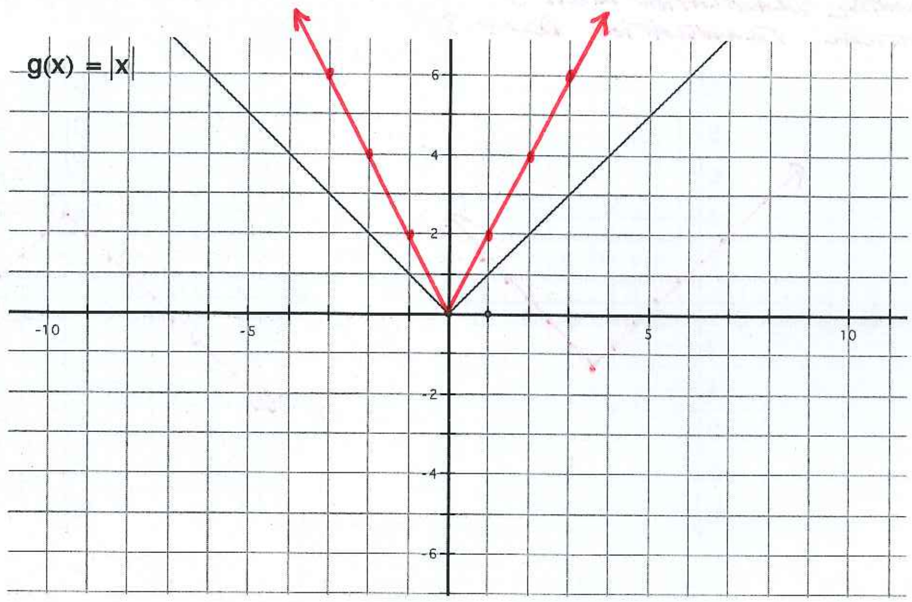
Example 3: Graph the function $y=|x|$ with each of these functions. How does the graph of each function compare to the original graph.

a. $\frac{y}{2}=|x|$ $y=2|x|$

X	Y
-3	6
-2	4
0	0
1	2
3	6

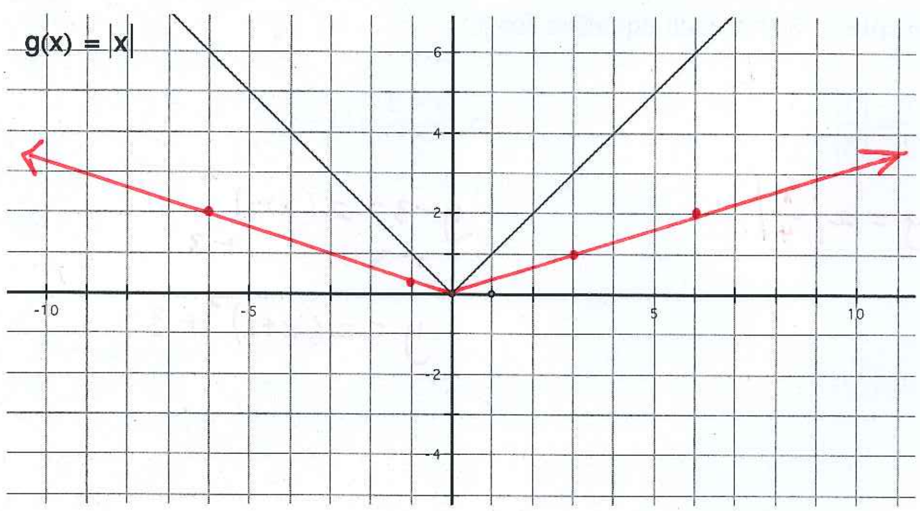
VERTICAL STRETCH SF 2



b. $y=\left|\frac{x}{3}\right|$

X	Y
-6	2
-1	$\frac{1}{3}$
0	0
3	1
6	2

HORIZONTAL STRETCH SF 3



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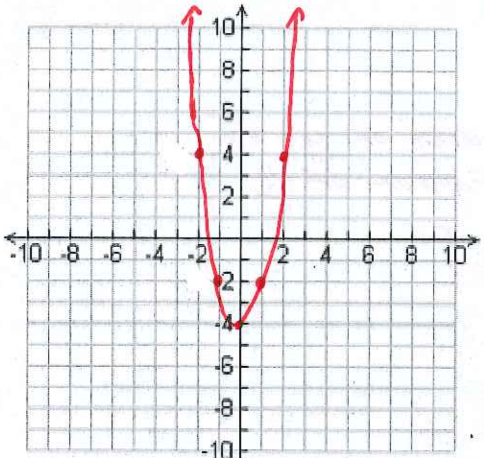
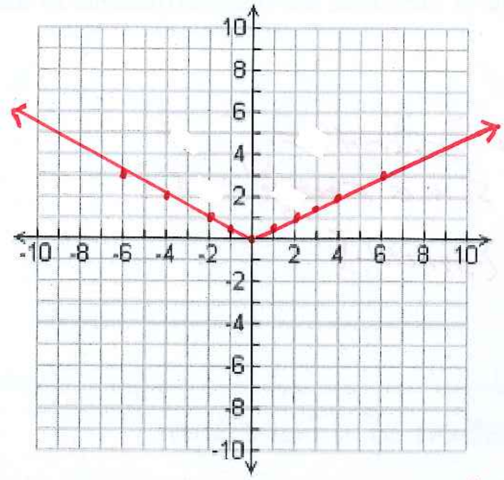
Example 5: Describe the transformation. Then graph the equation:

$y = 2x^2 - 4$



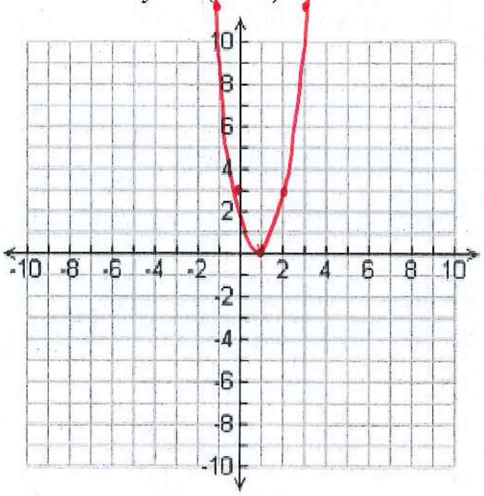
VERTICAL DILATION SF 2.
VERTICAL TRANSLATION DOWN 4.

$y = 0.5|x|$



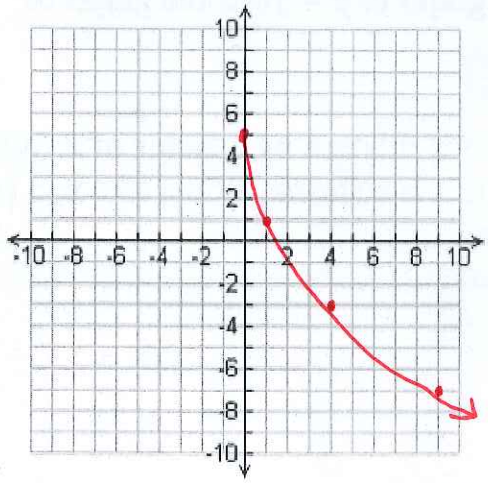
VERTICAL DILATION SF $\frac{1}{2}$

$y = 3(x-1)^2$



HORIZONTAL TRANSLATION RIGHT 2
VERTICAL DILATION SF 3

$y = -4\sqrt{x} + 5$



VERTICAL DILATION SF 4
VERTICAL REFLECTION ACROSS X-AXIS
VERTICAL TRANSLATION UP 5

(0,0) $0 \times 4 = 0 \to 0 \to 5$
(1,1) $1 \times 4 = 4 \to -4 \to 1$
(4,2) $2 \times 4 = 8 \to -8 \to -3$
(9,3) $3 \times 4 = 12 \to -12 \to -7$

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Scale factor A number that determines the amount by which a graph is dilated, either horizontally or vertically.

Example 4: Describe the transformations to the following equations:

a. $y = 3x^2$

VERTICAL DILATION OF $y = x^2$ BY A SCALE FACTOR OF 3.

{Stretch}

b. $y = 0.5\sqrt{x}$

VERTICAL DILATION OF $y = \sqrt{x}$ BY A SCALE FACTOR OF $\frac{1}{2}$.

{Shrink}

Dilation of a Function

A **dilation** is a transformation that expands or compresses a graph either horizontally or vertically.

Given the graph of $y = f(x)$, the graph of

$$\frac{y}{b} = f(x) \quad \text{or} \quad y = bf(x)$$

is a vertical dilation by a factor of b. When $|b| > 1$, it is a stretch; when $0 < |b| < 1$, it is a shrink. When $b < 0$, a reflection across the x-axis also occurs.

Given the graph of $y = f(x)$, the graph of

$$y = f\left(\frac{x}{a}\right)$$

is a horizontal dilation by a factor of a. When $|a| > 1$, it is a stretch; when $0 < |a| < 1$, it is a shrink. When $a < 0$, a reflection across the y-axis also occurs.

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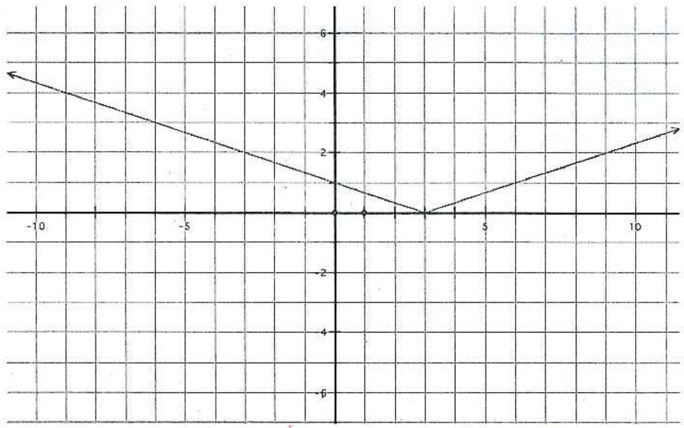
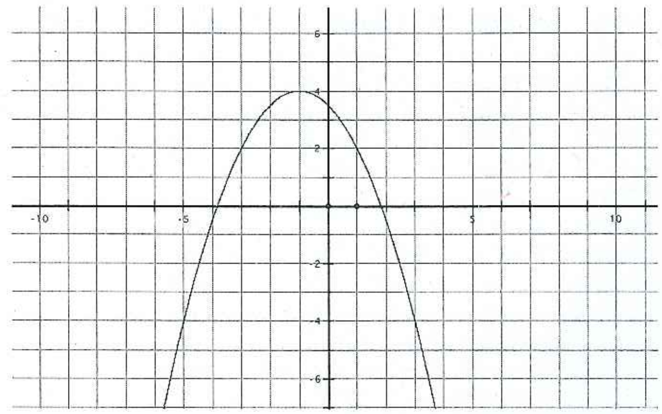
Write the equation of each function below.





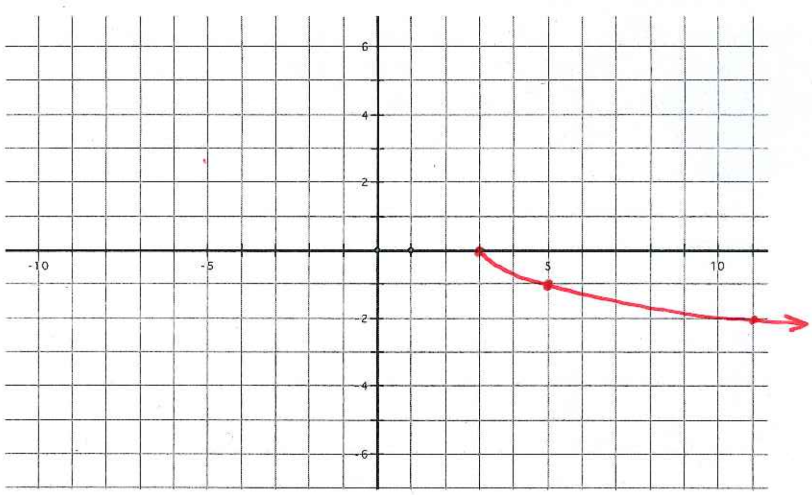
$y = \frac{1}{3}|x-3|$ RIGHT 3 · VER DIL SF $\frac{1}{3}$

$y = \left|\frac{1}{3}(x-3)\right|$ HOR DIL SF 3, RIGHT 3

$y = \left|\frac{1}{3}x - 1\right|$ RIGHT 1, HOR DIL SF 3

$y = -\frac{1}{2}(x+1)^2 + 4$ LEFT 1, REFL VERT, VD SF $\frac{1}{2}$, · UP 4

Graph the function $f(x) = -\sqrt{(\frac{x-3}{2})}$ *below.*



$f(x) = -\sqrt{\frac{1}{2}(x-3)}$

HD SF 2

RIGHT 3

VERT REFL

$(0,0) \to (0,0) \to (3,0) \to (3,0)$

$(1,1) \to (2,1) \to (5,1) \to (5,-1)$

$(4,2) \to (8,2) \to (11,2) \to (11,-2)$